Propagation

Ince-Gaussian Beams: The Third Family of Eigenmodes of Stable Laser Resonators

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Hermite–Gaussian beams (HGBs) and Laguerre–Gaussian beams (LGBs) have been studied extensively for more than 40 years.[1] Their numerous applications in science and engineering make them highly relevant and useful. HGBs and LGBs exhibit three important features: they are transverse eigenmodes of stable laser resonators; they constitute two complete families of exact and orthogonal solutions of the paraxial wave equation (PWE); and they do not change transverse shape on propagation, i.e., they are structurally stable. Because of the general relevance of Gaussian beams in optics, it is important to identify other orthogonal modes of stable laser resonators with no rectangular or circular symmetry.

In recent papers the existence of Ince–Gaussian beams (IGBs), which constitute the third complete family of transverse eigenmodes of stable resonators, was theoretically[2–4] and experimentally[5] demonstrated. These new modes are exact and orthogonal solutions of a PWE in elliptic coordinates and can be considered continuous transition modes between HGBs and LGBs. The transverse distribution of IGBs is described by Ince polynomials. Most of the propagating and resonating characteristics of HGBs and LGBs can be extended to IGBs. In addition to wavelength λ and waist size w_0 at plane $z = 0$, IGBs are characterized by two indices (p,m), the parity (even or odd), and a free continuous parameter ε that adjusts the ellipticity of the transverse shape of the beam.

Building on the theoretical research reported in Refs. 2 and 3, we have been able to produce IGBs experimentally with high fidelity.[5] To create IGBs, we used a self-built diode-pumped solid-state laser, the active medium of which was a Nd:YVO$_4$ crystal pumped at 808 nm. Output power was approximately $P_{\text{out}} = 20$ mW at an emission wavelength of $\lambda_0 = 1{,}064$ nm. To generate IGBs we slightly broke the symmetry of the resonator by shifting the output coupler sideways by several tens of micrometers. The intensity patterns shown in Fig. 1(a) and Ref. 5 provide, for the first time to our knowledge, experimental verification of this new class of beams. Note the excellent agreement with the theoretical patterns shown in Fig. 1(b). The patterns exhibit an inherent elliptical structure. Index m defines the number of hyperbolic nodal lines, and $(p - m)/2$ is the number of elliptic nodal lines.

A suitable superposition of even and odd IGBs with the same pair of indices (p, m) makes it possible to construct helical IGBs the phase of which rotates elliptically around the line that joins the foci of the ellipses; see Fig. 1(c). A field of this kind carries orbital angular momentum and exhibits multiple vortices. Both are attractive properties for potential applications in optical tweezers and particle trapping. Our results extend the fundamental theory of high-order Gaussian beams by adding the new IGBs to the well known HGBs and LGBs.

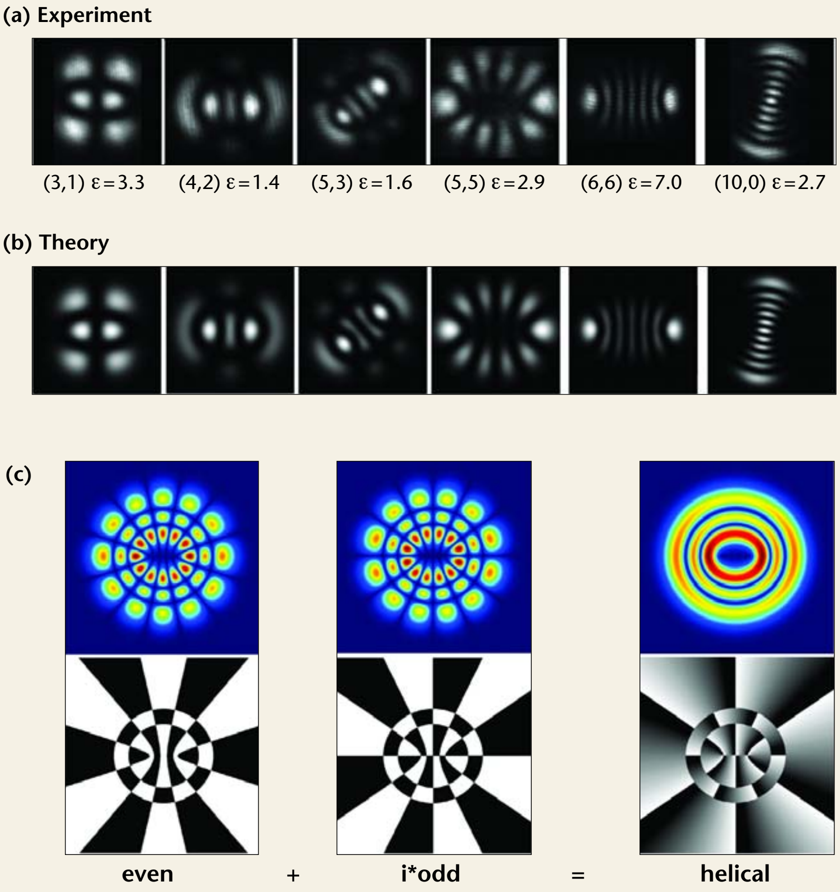

Figure 1. (a) Experimental and (b) theoretical intensity patterns of even IGBs. (c) Transverse amplitudes and phases of stationary even and odd IGBs ($p = 10$, $m = 6$) at the waist plane and the corresponding helical IGB.

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References

1. A. E. Siegman, Lasers (University Science, Mill Valley, Calif., 1986).
2. M. A. Bandres and J. C. Gutiérrez–Vega, Opt. Lett. **29**, 144–6 (2004).
3. M. A. Bandres and J. C. Gutiérrez–Vega, J. Opt. Soc. Am. A **21**, 873–80 (2004).
4. M. A. Bandres, Opt. Lett. **29**, 1724–6 (2004).
5. U. T. Schwarz et al., Opt. Lett. **29**, 1870–2 (2004).

Exact Unified Theory of Scalar Paraxial and Nonparaxial Beams

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Recent technological developments have made possible the construction of miniature structures of the order of several nanometers. Examples are microdisk lasers and nanolasers and the so-called "lab on a chip," which is a microchip with a built-in nanosized light source and sensors to perform instant and detailed analyses for chemistry, biology, and medical studies. At these dimensions, the transverse size of laser light beams can be of the order of a few wavelengths, and their main features might behave differently from what we are used to with paraxial beams. For example, electromagnetic vector effects start to become an important issue that could manifest itself in different ways.

There are two methods commonly used to describe nonparaxial beams, or beams which have a diameter of a few wavelengths. One uses the addition of correction terms to paraxial beam solutions and the other, the virtual point source method, assumes a point source placed in a complex space.[1,2] A drawback of the virtual source solutions is that they carry an inherent singularity that makes them inadequate to describe propagating fields near the origin or focal source point. To eliminate this problem, nonsingular spherical stationary waves have been used, but even these nonsingular solutions are problematic because infinite energy is required to realize them physically.

In a recent publication, we investigated the traveling-wave properties of the Helmholtz equation in spheroidal coordinates and found a precise unified formulation for beam propagation that is physically consistent for paraxial and nonparaxial regimes.[3]

By determining that the beam-width evolution of paraxial Gaussian modes follows a hyperbola, we found the relation between the physical parameters of Gaussian modes and those of nonparaxial spheroidal beam-like solutions. Then, by using the accepted criteria for the maximum angle that a paraxial beam can diffract, we defined a threshold between nonparaxial and paraxial beams.[4] As expected, within the paraxial limit the oblate spheroidal solutions investigated tend to the known Laguerre–Gaussian beams. By extension, a similar analysis of the two-dimensional Helmholtz equation yields paraxial Hermite–Gaussian modes from nonparaxial Mathieu modes.

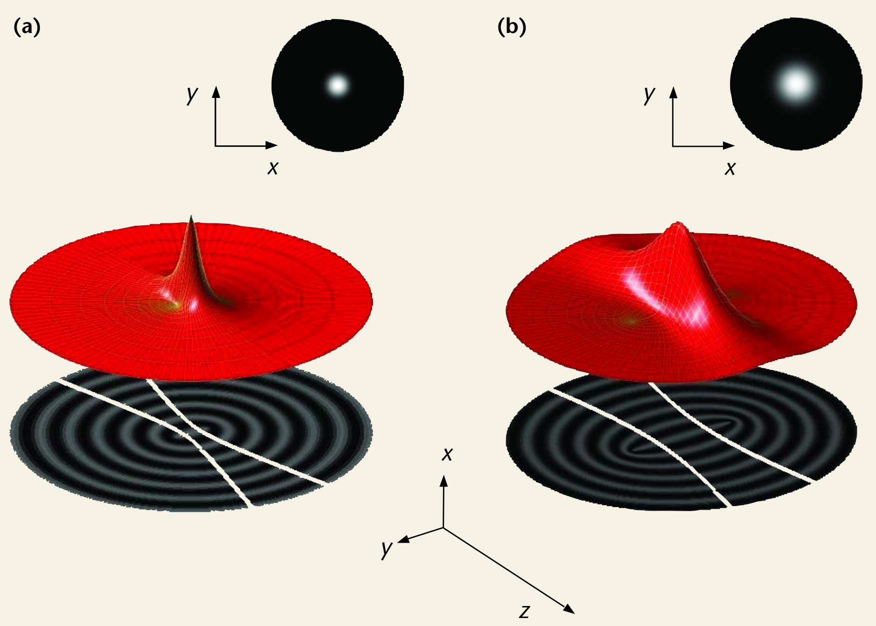

Figure 1. Zero-order spheroidal mode: (a) nonparaxial and (b) paraxial. The red surface plot corresponds to the intensity distribution, the bottom plot to the phase fronts. The beam width hyperbolic behavior is also shown. Each inset depicts the transverse profile at the beam waist.

Figure 1 shows the zero-order spheroidal scalar mode for (a) nonparaxial and (b) paraxial situations. The top surface plot represents the intensity distribution and the bottom density plot shows the structure of the wave fronts. The inset shows the transverse intensity at plane $z = 0$. In the paraxial case, when the intensity distribution is compared with the Laguerre–Gaussian beam solutions, the differences are negligible.

At this stage we have presented only the scalar solutions, however, it is well known that the vector solutions of the Helmholtz equation can be easily found after the scalar solutions are obtained. We expect that the traveling-wave beam-like solutions of the Helmholtz equation presented here shed new light on the physics of beam propagation of a few wavelengths. For example, paraxial and nonparaxial spheroidal beams are also structurally stable, i.e. shape invariant, in the same manner as the paraxial Laguerre–Gaussian beam.

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References

1. M. Lax et al., Phys. Rev. A **11**, 1365–70 (1975).
2. S. Y. Shin and L. B. Felsen, J. Opt. Soc. Am. **67**, 699–700 (1977).
3. G. Rodríguez-Morales and S. Chávez-Cerda, Opt. Lett. **29**, 430–2 (2004).
4. A. E. Siegman, Lasers, (University Science, Mill Valley, Calif., 1986).