The Faraday Effect

By Masud Mansuripur

In this issue of *Optics & Photonics News*, the Engineering column has been expanded to highlight one of the major technical achievements of Michael Faraday, a founding father of the field of electromagnetics.

Michael Faraday (1791–1867) was born in a village near London into the family of a blacksmith. His family was too poor to keep him in school and, at the age of 13, he took a job as an errand boy in a bookshop. A year later he was apprenticed as a bookbinder for a term of seven years. Faraday was not only binding the books but reading many of them, and they kindled in him a burning interest in science.

When his position in the bookshop was expiring, he applied for the job of assistant to Sir Humphry Davy, the celebrated chemist, whose lectures Faraday had attended during his apprenticeship. When Davy asked the advice of one of the governors of the Royal Institution of Great Britain about the employment of a young bookbinder, the man said: "Let him wash bottles! If he is any good he will accept the work; if he refuses, he is not good for anything." Faraday accepted, and remained with the Royal Institution for the next fifty years, first as Davy's assistant, then as his collaborator, and finally, after Davy's death, as his successor. It has been said that Faraday was Davy's greatest discovery.

In 1823 Faraday liquefied chlorine and in 1825 he discovered the substance known as benzene. He also did significant work in electrochemistry, discovering the laws of electrolysis. However, his greatest work was with electricity. In 1821 Faraday built two devices to produce what he called electromagnetic rotation, that is, a continuous circular motion from the circular magnetic force around a wire. Ten years later, in 1831, he began the great series of experiments which led him to the discovery of electromagnetic induction. These experiments form the basis of modern electromagnetic technology.

Apart from numerous publications in scientific magazines, the most remarkable document pertaining to his studies is the diary he kept continuously from 1820 to 1862. (This was published in 1932 by the Royal Institution in seven volumes containing a total of 3,236 pages, with a few thousand marginal drawings.) Queen Victoria rewarded Faraday's lifetime of achievement by granting him the use of a house at Hampton Court and a knighthood. Faraday accepted the cottage but gracefully rejected the knighthood.[1]

On 13 September, 1845 Faraday discovered the magneto-optical effect that bears his name. This day's entry in his diary reads: "Today worked with lines of magnetic force, passing them across different bodies (transparent in different directions) and at the same time passing a polarized ray of light through them and afterwards examining the ray by a Nichol's Eyepiece or other means." After describing several unsuccessful attempts in which the ray of light was passed through air and several other substances, Faraday wrote in the same day's entry: "A piece of heavy glass which was 2 inches by 1.8 inches, and 0.5 of an inch thick, being silico borate of lead, and polished on the two shortest edges, was experimented with. It gave no effects when the same magnetic poles or the contrary poles were on opposite sides (as respects the course of the polarized ray)—nor when the same poles were on the same side, either with a constant or intermitting current—BUT, when contrary magnetic poles were on the same side, there was an effect produced on the polarized ray, and thus magnetic force and light were proved to have relation to each other. This fact will most likely prove exceedingly fertile and of great value in the investigation of both conditions of natural force."

Electromagnetic basis of the Faraday effect

Magneto-optical (MO) effects are best described in terms of the dielectric tensor ε of the medium in which the interaction between the light and the applied magnetic field (or the internal magnetization of the medium) takes place.[2]

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$$\varepsilon = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$

In an isotropic material (such as ordinary glass) the three diagonal elements of ε are identical, and in the presence of a magnetic field along the Z-axis, there is a non-zero off-diagonal element ε', which couples the x- and y-components of the optical E-field, that is,

$$\varepsilon = \begin{pmatrix} \varepsilon & \varepsilon' & 0 \\ -\varepsilon' & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}$$

In general, ε and ε' are wavelength-dependent, but over a narrow range of wavelengths they might be treated as constants. In a transparent material, where there is no optical absorption, ε is real and ε' is imaginary. However, in the most general case of an absorbing MO material both ε and ε' may be complex numbers. For dia- and paramagnetic media, ε' is proportional to the applied magnetic field H, while for ferro- and ferrimagnetic materials the spin-orbit coupling is the dominant source of the MO interaction, making ε' proportional to the magnetization M of the medium.[2] Since $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$ (in CGS units), the B-field inside the medium may be denoted as the source of the MO effects.

When a polarized beam of light propagates in a medium along the direction of the magnetic field **B**, the material interacts with the right- and left-circularly polarized (RCP and LCP) components of the beam with different refractive indices, $n^{\pm} = (\varepsilon \pm i\varepsilon')^{1/2}$. For fused silica glass at the wavelength of $\lambda = 550$ nm, for example, $\varepsilon \approx 2.25$ and $\varepsilon' \approx 10^{-7}$i per kOe of applied magnetic field. (Note that both n^+ and n^- in this case are real-valued and, therefore, there is no absorption.) For linearly polarized light passing through a length L of the material under the influence of a B-field, the two circular polarization components suffer a relative phase-shift $\Delta\phi = 2\pi L\,(n^+ - n^-)/\lambda$.[3,4] As shown in Figure 1, a change of relative phase between the RCP and LCP components is equivalent to a rotation of the plane of polarization by the Faraday angle $\theta_F = \frac{1}{2}\Delta\phi$. In the above example, $\theta_F \sim 0.22°$ at $\lambda = 550$ nm for a 1 cm-thick slab immersed in a 1 kOe magnetic field. The figure of 0.22°/cm/kOe is known as the Verdet constant of fused silica at the specified wavelength.[3]

Certain magnetic materials (e.g., magnetic garnets) are transparent enough to transmit a good fraction of the light while producing a fairly large Faraday rotation. These materials can be magnetized in a given direction and sustain their magnetization when the external field is removed. Therefore, the Faraday effect in these media may be observed in the absence of an external magnetic field. At $\lambda = 550$ nm, for instance, a typical crystal of bismuth-substituted rare-earth iron garnet may have $\varepsilon \approx 5.5 + 0.025i$ and $\varepsilon' \approx 0.002 - 0.01i$. The complex refractive indices for RCP and LCP light are thus $(n + ik)^+ \approx 2.347 + 0.006i$ and $(n + ik)^- \approx 2.343 + 0.005i$, yielding a Faraday rotation angle $\theta_F \approx 1.3°$ for a micron-thick slab of this crystal. The absorption coefficient of the material α equals $4\pi kL/\lambda$, where k is the imaginary part of the complex refractive

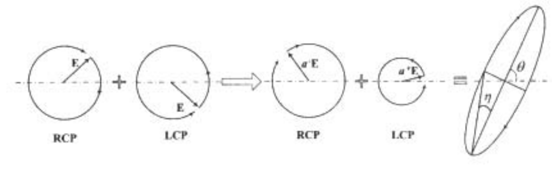

Figure 1. A linearly polarized beam of light may be considered as the superposition of equal amounts of right- and left-circularly polarized beams. In going through a perpendicularly magnetized slab of material at normal incidence, the two components of circular polarization experience different (complex) refractive indices and, therefore, each emerges from the medium with a different phase and amplitude. The amplitudes of the emergent beams may be denoted by a^+ and a^-, and their phase difference by $\Delta\phi$. The superposition of the emergent circular polarization states yields elliptical polarization. The angle of rotation of the major axis of the ellipse from the horizontal direction (which is the direction of the incident linear polarization) is given by $\theta = \frac{1}{2}\Delta\phi$, and the ellipticity η is given by $\tan\eta = (a^+ - a^-)/(a^+ + a^-)$.

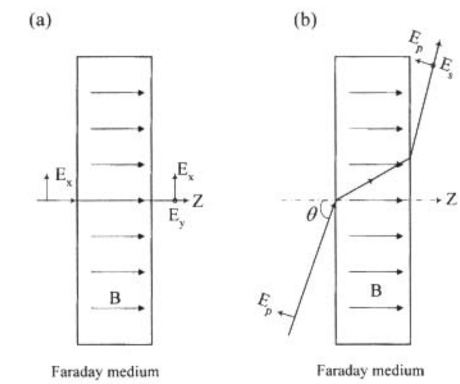

Figure 2. Faraday effect in the polar geometry. (a) In going through a slab of magnetic material, a linearly polarized beam of light with its E-field along the X-axis acquires a component of polarization along Y. The lines of B-field shown within the medium represent either an externally-applied magnetic field or the intrinsic magnetization of the medium. (b) The effect is also observed at oblique incidence. Shown here is a p-polarized incident beam, which acquires an s-component upon transmission through the magnetic medium. (If the incident beam were s-polarized, the magneto-optically induced polarization would have been in the p-direction.) In general, upon reversing the B-field from +Z to –Z direction, the magneto-optically induced component of polarization changes sign.

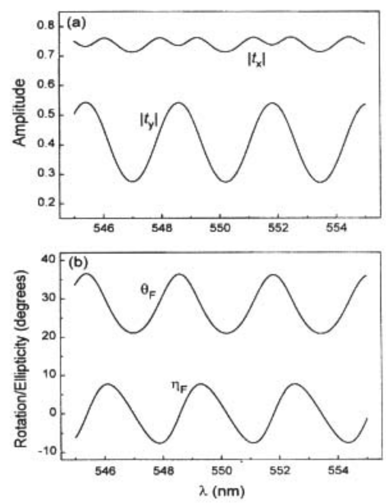

Figure 3. A plane wave, linearly polarized along the X-axis, is normally incident on a 20 μm-thick slab, as shown in Figure 2(a). The slab ($\varepsilon = 5.5$, $\varepsilon' = 0.01i$) is magnetized along the Z-axis. (a) Plots of $|t_x|$ and $|t_y|$, the transmitted polarization components along the X- and Y-axes, as functions of λ. (b) Plots of polarization rotation angle θ_F and ellipticity η_F versus λ.

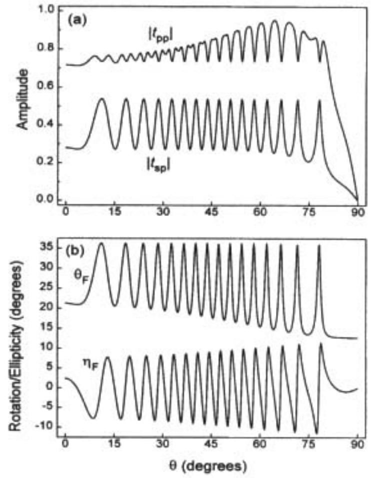

Figure 4. A p-polarized plane wave (λ = 550 nm) is incident at oblique angle θ on a 20 μm-thick slab, as shown in Figure 2(b). The slab ($\varepsilon = 5.5$, $\varepsilon' = 0.01i$) is magnetized along the Z-axis. (a) Plots of $|t_{pp}|$ and $|t_{sp}|$, the transmitted polarization components along the p- and s-directions, as functions of θ. (b) Plots of θ_F and η_F versus θ.

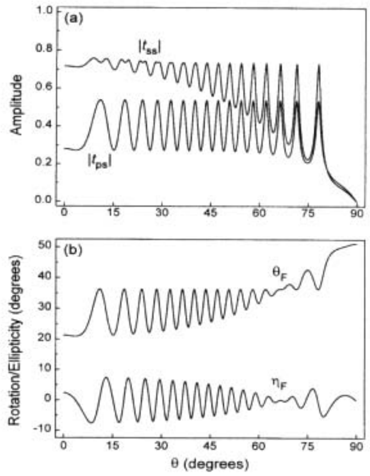

Figure 6. Same as Figure 4, except that the incident beam in the present case is s-polarized.

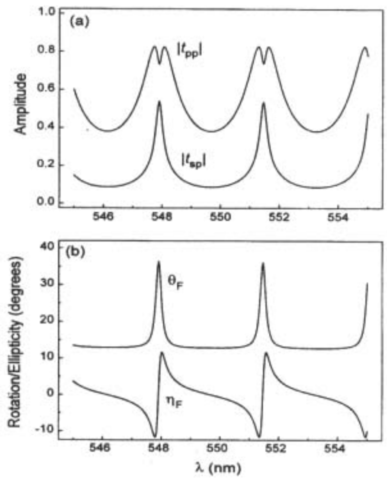

Figure 5. A p-polarized plane wave is incident at θ = 85° on the slab described in Figure 4. (a) Plots of $|t_{pp}|$ and $|t_{sp}|$, the transmitted p- and s-components of polarization, as functions of λ. (b) Plots of θ_F and η_F versus λ.

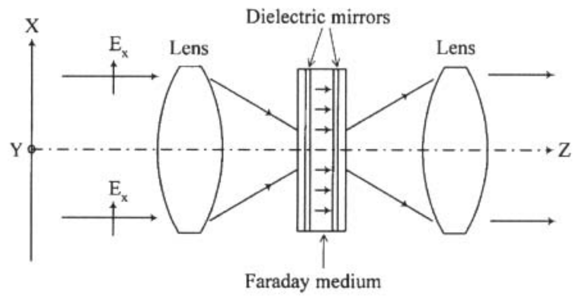

Figure 7. A Faraday medium in a Fabry-Perot resonator is placed in a convergent cone of light. The incident plane wave is linearly polarized along the X-axis, and the 0.8NA focusing lens is free from aberrations. The 20 μm-thick Faraday medium ($\varepsilon = 5.5$, $\varepsilon' = 0.01i$) is uniformly magnetized along the Z-axis. The mirrors coated on the front and back facets of the Faraday slab each consist of 10 alternating layers of high-index ($n = 2$) and low-index ($n = 1.5$) quarter-wave-thick dielectrics. The collimating lens is identical to the focusing objective, and the emergent beam is observed at the exit pupil of the collimator.

index. For the above garnet, therefore, $\alpha \approx 0.12$ per micron, which is equivalent to 1dB loss of light for every 2 μm of crystal thickness. In other words, this garnet delivers 2.6° of polarization rotation per dB of loss. These crystals can be grown in a range of thickness from a fraction of a micron to about 100 μm. Thicker crystals are useful at longer wavelengths, where the losses are small, but the Faraday rotation generally decreases with the increasing wavelength as well.

Faraday rotation in a transparent slab

For the sake of simplicity we ignore the effects of absorption in the Faraday medium and consider a transparent slab of magnetic material having a real ε and a purely imaginary ε'. Thus, consider a 20 μm-thick slab having $\varepsilon = 5.5$, $\varepsilon' = 0.01i$. The slab is magnetized perpendicular to the plane of its surface, and a linearly polarized beam of light (E-field along the X-axis) is sent at normal incidence through the slab, as in Figure 2(a).[5, 6] Real sources of light, of course, are never perfectly monochromatic and, therefore, we assume a finite spectral bandwidth for the light source, covering the range λ = 545 nm – 555 nm. Figure 3 shows computed plots of the transmitted amplitudes, $|t_x|$ and $|t_y|$, as well as the polarization rotation and ellipticity angles, θ_F and η_F, versus λ.[7] Due to multiple reflections at the front and rear facets of the slab, these functions vary periodically with λ. (The same interference phenomena are responsible for the non-zero values of η_F, which would otherwise be absent in a transparent medium.) The net Faraday rotation angle is the average value of θ_F over the relevant range of wavelengths, but one should also recognize that the wavelength-dependence of the direction of emergent polarization produces a certain amount of depolarization in the emergent beam. The Faraday rotation combined with the spectral bandwidth of the light source thus causes partial depolarization as a direct consequence of interference among the multiple reflections.

Oblique incidence

Figure 4 shows the transmitted amplitudes and polarization angles versus the angle of incidence θ in the case of a 20 μm-thick slab magnetized along the Z-axis ($\varepsilon = 5.5$, $\varepsilon' = 0.01i$) when, as shown in Figure 2(b), a p-polarized plane wave having the single wavelength of $\lambda = 550$ nm is incident on the slab.[7] The oscillations in the transmitted amplitudes/polarization angles are caused by interference among the multiply reflected beams at the facets of the slab. Aside from the interference effects, however, note that the Faraday effect does not show any signs of abatement with the increasing angle of incidence. The reason is that, while the direction of propagation of the beam increasingly deviates from the direction of the B-field, the propagation distance simultaneously increases, keeping the net interaction between the magnetic material and the beam of light at a constant level.

Figure 5 shows the case of oblique incidence at $\theta = 85°$ on the same slab in the range $\lambda = 545$ nm – 555 nm.[7] As in the case of normal incidence depicted in Figure 3, we note a significant variation of the Faraday angles/amplitudes within this narrow range of wavelengths. Although the beam inside the slab travels at ~ 25° relative to the direction of magnetization of the material, the maximum Faraday effect as exemplified by $|t_{sp}|$ is the same as in normal incidence, because the propagation distance is correspondingly adjusted. The wavelength-averaged Faraday rotation may be lower at larger angles of incidence, but this is just a consequence of interference; it is not caused by the reduction of the intrinsic optical activity of the slab. If, for instance, the facets are anti-reflection coated, or if the beam enters and exits through index-matched spherical surfaces, then multiple reflections would be eliminated and the Faraday rotation becomes independent of the incidence angle.

The above discussions were confined to the case of p-polarized incident beam, but the conclusions remain valid for s-polarized light as well. For example, Figure 6 is the counterpart of Figure 4, showing the transmitted amplitudes and polarization angles versus the angle of incidence for an s-polarized incident beam.[7] Note that the magneto-optically generated component of polarization t_{ps} in Figure 6 is identical to t_{sp} in Figure 4. This is an important and completely general result, indicating that the amount of light converted from one polarization to another is independent of the state of incident polarization.

Faraday medium in a Fabry-Perot resonator

Because the Faraday effect is amplified as a result of the beam propagating back and forth within a magnetized medium, it is interesting to observe the enhancement of the Faraday effect in a Fabry-Perot resonator. Figure 7 shows the diagram of a system that may be used to monitor such enhancement in a range of angles of incidence. The first objective lens (NA = 0.8) focuses a linearly polarized beam of light onto the Fabry-Perot resonator, and the second, identical lens collimates the transmitted beam, thus allowing observation at the exit pupil. For a 20 μm-thick slab of transparent magnetic material sandwiched between a pair of dielectric mirrors, Figure 8 shows the computed patterns of intensity and polarization angles at

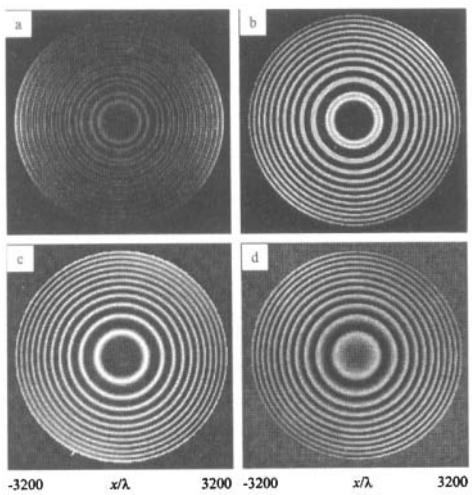

Figure 8. Intensity and polarization patterns in the exit pupil of the collimating lens of Figure 7. (a) Intensity distribution of the emergent X-polarized component. The bright rings indicate the regions where the conditions of resonance are met and the light passes through the resonator. (b) Intensity distribution of the emergent Y-polarized component. The bright rings coincide with those in (a), indicating that the conditions of resonance for the incident polarization are the same as those for the magneto-optically induced polarization. (c) Polarization rotation angle θ_F of the emergent beam encoded in gray-scale. The range of values of θ_F is −23° (black) to +63° (white). (d) Polarization ellipticity η_F of the emergent beam encoded in gray-scale. The range of values of η_F is −32° (black) to +42° (white).

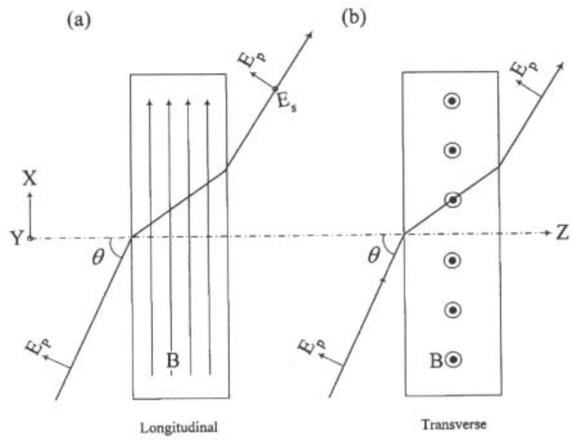

Figure 9. (a) Longitudinal Faraday effect is observed when the direction of the B-field within the slab of material is parallel both to the surface of the slab and to the plane of incidence. The rotation of polarization in this case occurs only at oblique incidence, where, upon transmission, a p-polarized beam acquires an s-component and vice versa. If the direction of **B** is reversed, the magneto-optically induced component of polarization will change sign. (b) The transverse effect occurs when the B-field lies in the plane of the sample perpendicular to the plane of incidence. The MO interaction in this case occurs only when the incident beam is p-polarized. Even then there is no polarization rotation, only a change in the magnitude of the B-field causes a slight change in the magnitude of the transmitted p-light. The transverse effect is small and is not bipolar, meaning that reversing the direction of **B** does not affect the emergent beam.

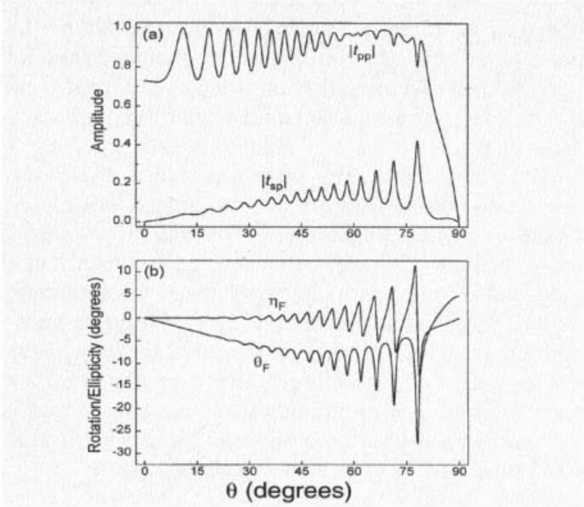

Figure 10. Longitudinal Faraday effect, arising when a p-polarized plane wave (λ = 550 nm) is incident at oblique angle θ on a 20 μm-thick slab. The slab ($\varepsilon = 5.5$, $\varepsilon' = 0.01i$) is magnetized along the X-axis, as depicted in Figure 9(a). (a) Transmitted amplitudes $|t_{pp}|$ and $|t_{sp}|$ versus θ. (b) Polarization rotation angle θ_F and ellipticity η_F versus θ.

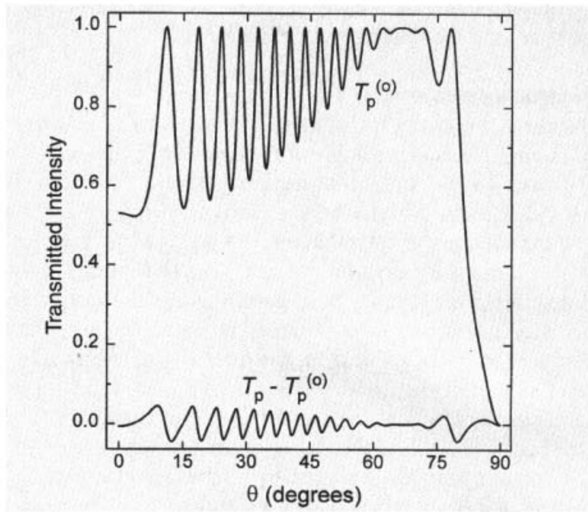

Figure 11. Transverse Faraday effect, arising when a p-polarized plane wave (λ = 550 nm) is incident at oblique angle θ on a 20 μm-thick slab. The slab ($\varepsilon = 5.5$) is magnetized along the Y-axis, as shown in Figure 9(b). In the absence of the B-field, $\varepsilon' = 0$, and the transmission of the slab for a p-polarized incident beam is denoted by $T_p^{(0)}$. When a strong B-field is introduced (corresponding to $\varepsilon' = 0.1i$ in this case), the transmission changes to T_p. Shown here is the transmission differential $\Delta T_p = T_p - T_p^{(0)}$ as function of θ.

the exit pupil of the collimator.[7] This figure indicates that the rings of maximum transmission also correspond to locations of maximum polarization rotation. The maximum and minimum rotation angles in Figure 8(c) are +63° and −23°, respectively, well in excess of the rotations obtained from the bare slab. Also note in Figures 8(c, d) the asymmetrical nature of the polarization angles in the first and third quadrants on the one hand and in the second and fourth quadrants on the other.

Longitudinal and transverse geometries

When the direction of the B-field is in the plane of the slab as well as in the plane of incidence, as in Figure 9(a), one observes the longitudinal Faraday effect. In this case ε' occupies the position of ε_{yz} in the dielectric tensor. The transverse effect occurs when the B-field, while in the plane of the sample, is perpendicular to the incidence plane, as in Figure 9(b). In this case ε' occupies the position of ε_{xz}.

In the longitudinal case at normal incidence no polarization rotation occurs, but the effect begins to show up with an increasing angle of incidence. For a p-polarized plane wave (λ = 550 nm) obliquely incident on a 20 μm-thick slab of magnetic material ($\varepsilon = 5.5$, $\varepsilon' = 0.01i$), Figure 10 shows the computed amplitudes of the transmitted p- and s-polarized light as well as the angles of rotation and ellipticity versus the incidence angle θ. One could readily compute similar results for an s-polarized incident beam as well. In both cases the MO effect is bipolar, meaning that a reversal of the direction of the B-field reverses the signs of θ_F and η_F. Moreover, as in the polar case discussed earlier, the magneto-optically generated component of polarization turns out to be the same for both directions of incident polarization, that is, $t_{sp} = t_{ps}$.

The transverse effect is very different from both polar and longitudinal effects. With s-polarized incident light, where the optical E-field is parallel to the direction of the B-field in the slab, there is no MO effect whatsoever, but for p-polarized light the medium exhibits an effective refractive index $n = [\varepsilon + (\varepsilon'^2/\varepsilon)]^{1/2}$. Thus in the transverse case neither s- nor p-polarized beams undergo polarization rotation, but the magnitude of the transmitted p-light shows a weak dependence on magnetization, that is, $T_p = |t_p|^2$ becomes a function of the strength of the B-field. The transverse effect is not bipolar, so changing the direction of the B-field from +Y to −Y does not alter the magnitude of T_p. For a 20 μm-thick slab of transparent material with a fairly large MO coefficient ($\varepsilon = 5.5$, $\varepsilon' = 0.1i$) Figure 11 shows computed plots of $T_p^{(0)}$ (i.e., transmission in the absence of the B-field, when $\varepsilon' = 0$) and $\Delta T_p = T_p - T_p^{(0)}$ versus the angle of incidence θ.[7] Note, in particular, that $\Delta T_p \approx 0$ around the Brewster angle $\theta_B = 66.9°$, where vanishing surface reflectivity results in minimal interference effects.

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7. The simulations reported in this article were performed by MULTILAYER™ and DIFFRACT™. Both programs are products of MM Research, Inc., Tucson, AZ.

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