

Light pulses can be made  
to propagate with group  
velocities exceeding  
the speed of light in a  
vacuum or, at the  
opposite extreme,  
to come to a  
complete  
stop.

# *Fast Light,* Slow Light

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The phase velocity of light,  $c/n$ , can be greater than  $c$ , the speed of light in vacuum, when the refractive index  $n$  is less than 1, as in the case of x rays in glass. It is well known that a phase velocity greater than  $c$  does not contradict the special theory of relativity, because phase velocity is not a *signal* velocity. A signal requires a departure from monochromaticity and therefore cannot be characterized by a single-frequency quantity such as phase velocity.

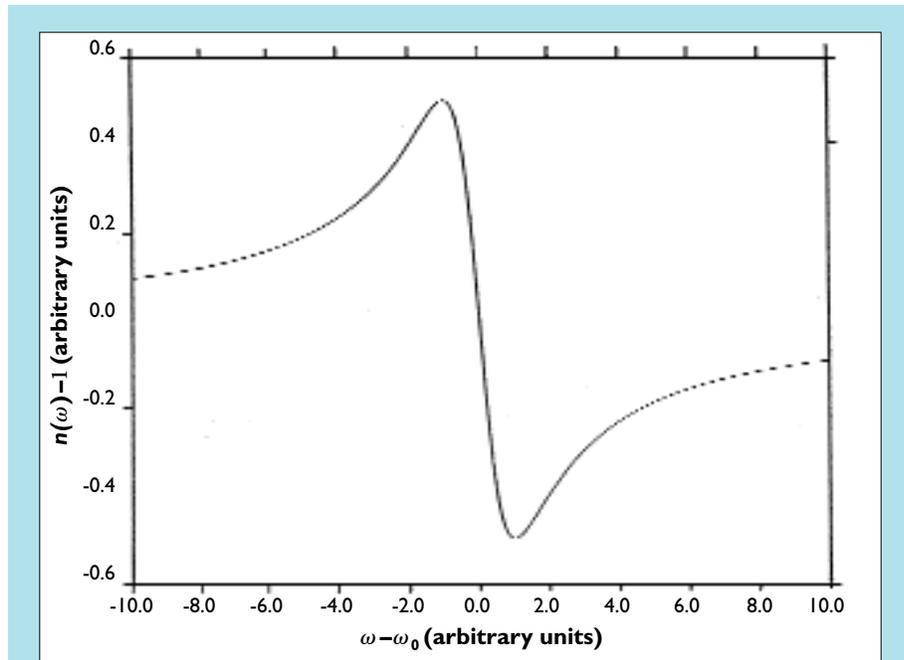
Consider the superposition of two monochromatic plane waves of the form  $\cos[(\omega \pm \delta\omega)t - (k \pm \delta k)z]$ . Addition gives  $2 \cos[\delta\omega(t - (\delta k/\delta\omega)z)] \cos(\omega t - kz)$ , which is a plane wave with an amplitude that retains its shape as it propagates with the velocity  $\delta\omega/\delta k$ . More generally, if we superpose many waves whose frequencies and wave numbers are distributed in a small range about  $\omega$  and  $k$ , we obtain a wave of the form  $A(t - z/v_g) \cos(\omega t - kz)$ , where the *group velocity*  $v_g$  is defined as  $d\omega/dk$  (Ref. 1). The intensity, averaged over a few optical periods, is proportional to  $A^2(t - z/v_g)$ , which describes a pulse of light propagating without change of shape at the velocity  $v_g$ . Using the relation  $k = n(\omega)\omega/c$ , we obtain

$$v_g = \frac{c}{n + \omega dn/d\omega}, \quad (1)$$

where  $n$  and  $dn/d\omega$  are evaluated at the carrier frequency  $\omega$ . If the range of frequencies in the field is too broad, or if the medium is such that  $v_g$  varies too rapidly with frequency, the situation becomes more complicated and we must account for “group velocity dispersion.” But such complications can often be ignored.

Equation (1) shows that the group velocity can exceed  $c$  when  $dn/d\omega$  is negative, i.e., when the refractive index decreases with frequency. Since the refractive index normally increases with frequency in the visible, the case  $dn/d\omega < 0$  is called, for historical reasons, “anomalous dispersion.” Anomalous dispersion occurs when  $\omega$  is close to an absorption frequency of the medium (Fig. 1). In the case of an *amplifying* medium, the curve of  $n(\omega) - 1$  vs.  $\omega$  is simply reversed in sign compared with the curve of Fig. 1, and the dispersion becomes “anomalous” when  $\omega$  is tuned *away* from the center of the spectral line.

A group velocity greater than  $c$  does not contradict relativity because group velocity, like phase velocity, is not in general



**Figure 1.** Typical variation of  $n(\omega) - 1$  vs.  $(\omega - \omega_0)/\gamma$ , where  $\omega_0$  is a frequency corresponding to an absorption line in the medium of propagation and  $\gamma$  is the width of the absorption line. Near  $\omega_0$ ,  $n(\omega)$  exhibits anomalous dispersion, i.e., it decreases with  $\omega$ .

a *signal* velocity. Sommerfeld and Brillouin<sup>2</sup> defined a signal as a train of oscillations that starts from zero at some instant, and proved that such a signal cannot propagate faster than  $c$ . The group velocity characterizes the propagation of the “bulk” and the peak of a pulse, but not the point of discontinuity that starts from zero at some instant and describes the wave *front*. In fact, since this front is associated with infinite frequencies, and the refractive index approaches unity as the frequency approaches infinity, the *front velocity* is exactly  $c$ .

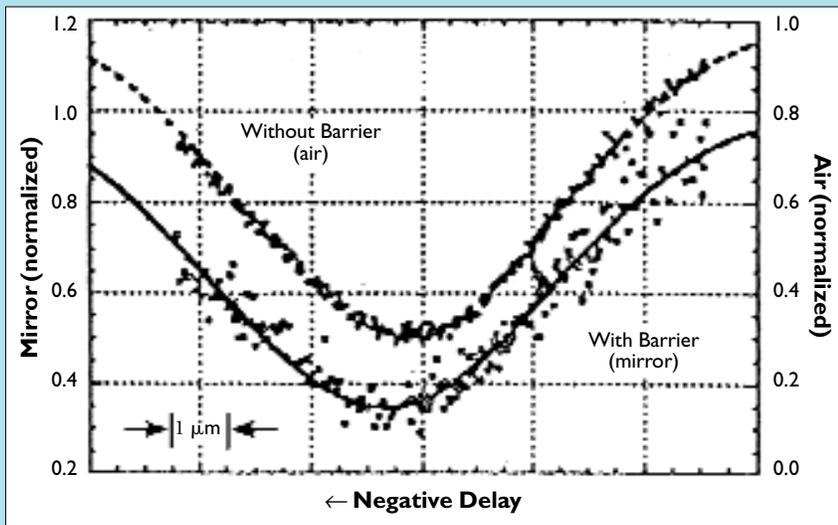
There has been an unfortunate, but widespread, tendency to imply that group velocity is synonymous with signal velocity. To cite just one example: a calculation of the right-hand side of Eq. (1) for the case of x rays in glass gives a velocity less than  $c$ , and it is concluded that, while the phase velocity can exceed  $c$ , the signal velocity cannot.<sup>3</sup> But the same identification of group velocity with the velocity of signal propagation in the case of anomalous dispersion would imply that a signal *can* propagate faster than  $c$ !

Many readers have on their shelves some classic texts on optics and electromagnetism in which it is incorrectly implied that a group velocity greater than  $c$  would contradict the special theory of rel-

ativity. Such a violation does not occur, it is argued, because when  $v_g > c$  a pulse becomes so severely distorted that group velocity is no longer a meaningful concept. However, experiments cited below demonstrate that group velocity can exceed  $c$  while remaining meaningful—and measurable—as the velocity of an undistorted pulse.

Equation (1) implies that the group velocity can be infinite or even negative if  $dn/d\omega$  is sufficiently negative. An infinite group velocity means that the peak of the pulse emerging from the medium occurs at the same time as the peak of the pulse entering the medium; the peak appears to cross the medium instantaneously. A negative group velocity means that the peak of the emerging pulse occurs at an earlier time than the peak of the incident pulse. Such “anomalous” group velocities can occur without significant pulse distortion. Negative and infinite group velocities have been observed, albeit with substantial pulse attenuation and some pulse compression.<sup>4</sup>

Anomalous group velocities have been pondered for many years in an entirely different context: particle tunneling in quantum mechanics. It was shown by MacColl in 1932 that the peak of a particle wave packet can tunnel across a potential barrier



**Figure 2.** Measured coincidence rate vs. path difference of twin photons, one passing through a tunnel barrier and the other through air. (From Ref. 6.)

er without any appreciable time delay, i.e., the group velocity of the wave packet can exceed  $c$ . This feature of quantum tunneling has been the subject of many papers and controversies, and more recently it has been observed in experiments at Berkeley<sup>5</sup> that demonstrated anomalous group velocities of single-photon wave packets.

In those experiments, a laser beam incident on a crystal was used to generate pairs of pulses having the same central wavelength and bandwidth (about 20 fs), each pulse of the wave packet corresponding to a single photon. One of the photons of each pair passed through air, whereas the other was incident on a dielectric consisting of alternating layers of high and low  $n$  along the direction of propagation. The wavelength was such that transmission through the dielectric stack could occur only in the form of an exponentially decaying, “evanescent” wave. In other words, a photon could pass through only by a (low-probability) tunneling process. To measure the femtosecond-scale delays between photons that traversed the tunnel barrier and their twins that passed through air, use was made of a “Hong-Ou-Mandel interferometer” in which each photon arrives at one of two detectors according to whether it is reflected or transmitted by a 50/50 beam splitter. If the two photons arrive simultaneously at the beam splitter, there is a destructive interference effect such that there is a zero probability of counting photons

simultaneously. This provides a sensitive measure of the path delays for the two photons, making it possible to determine the photon tunneling times. Figure 2 shows measured photon coincidence rates with and without the tunnel barrier in place. The negative delay measured with the barrier means that a single photon tends to pass through the barrier faster than it would propagate through an equal distance in air. Effective tunneling velocities of about  $1.7c$  were inferred from the measured photon coincidence rates.

Distortionless pulse propagation with group velocity greater than  $c$  and with a relatively small change in amplitude has recently been reported.<sup>7</sup> In these experiments, a gain doublet is employed, so that in the spectral region between two frequencies where the medium amplifies light, there is strong anomalous dispersion but little amplification or absorption. It was observed that the peak of a transmitted pulse appears at the end of the medium before the peak of the incident pulse appears at the entrance to the medium, i.e., the group velocity is negative. The transmitted peak occurred 62 ns before the incident peak, or in other words, the transmitted peak travels about 18 m ( $62 \text{ ns} \times c$ ) before the incident peak even arrives.

These experiments in no way contradict the principle of Einstein causality: the principle that no *signal* can propagate faster than  $c$ . It follows from Maxwell’s

equations that the transmitted pulse at time  $t$  is determined by the incident pulse at times  $t < L/c$ , where  $L$  is the distance of propagation (Fig. 3). The peak of the transmitted pulse is separated in time from the peak of the incident pulse by  $L/v_g$  and, as can be seen in Fig. 3, is not causally related to the peak of the incident pulse. In other words, the “superluminal” peak (group) velocity of the pulse is not the velocity with which a signal can be transmitted.

The larger the group velocity compared with  $c$ , the smaller the tail of the incident pulse that determines the transmitted pulse. The shaded portion of the transmitted pulse in Fig. 3 does not contain any information that is not already contained in the shaded portion of the incident pulse. The relation between the shaded areas of Fig. 3 can be cast in terms of a Taylor series expansion of the incident pulse amplitude, and this expansion is possible up to a time at which the incident pulse amplitude or one of its derivatives has a discontinuity. The point of discontinuity behaves as a Sommerfeld-Brillouin front, which propagates at the velocity  $c$ . Since it is only by such a discontinuity that new information can be carried by the incident pulse (for instance, it must switch from “off” to “on” at some time), it can be concluded that no signal, in the sense of new information, can propagate faster than  $c$ .

These propagation effects can also be described rather generally in terms of phasors. In this description, we regard the Fourier component  $A \exp [j(\omega t + \theta)]$  of a pulse as a vector (phasor) of length  $A$ , pointing from the origin of the complex plane and making an angle  $\omega t + \theta$  with respect to the real axis at time  $t$ . The peak of a Gaussian wave packet, for instance, corresponds to a time at which the phasors add up to produce the maximum amplitude, while at the early tail the phasors add up to produce nearly total destructive interference (Fig. 4). Propagation in a dispersive medium causes the phasors to precess. In the case of normal dispersion, the result is a retardation of the wave packet with respect to propagation in vacuum; the nearly closed polygon at point  $A$  in Fig. 4 unfolds with propagation and the phasors become aligned at the time  $A'$ . In the case of anomalous dispersion, however, the precession of the phasors proceeds in the opposite sense and results in an *advance* of the wave packet with respect to vacuum propagation, corresponding to a

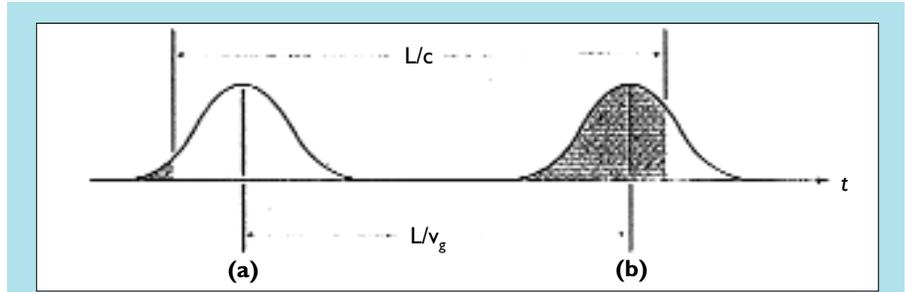
group velocity greater than  $c$ . The phasors ahead of a Sommerfeld-Brillouin wave front, on the other hand, can never combine to give anything but zero, since they all have zero length. This implies that no signal can propagate faster than the front velocity which, as we have already noted, is  $c$ .

While group velocities greater than  $c$  are not in conflict with relativity, they do suggest that the term “signal” needs to be better defined, especially when imperfect detectors and quantum effects are considered. For instance, it can be seen from Fig. 3 that, if  $v_g$  is much larger than  $c$ , essentially the entire transmitted pulse is a “re-construction” of a tiny, early-time tail of the incidence pulse. But this tail might be so small in amplitude that it is dominated by quantum fluctuations, which means that the transmitted pulse must be dominated by quantum fluctuations. Experiments thus far have not probed this regime.

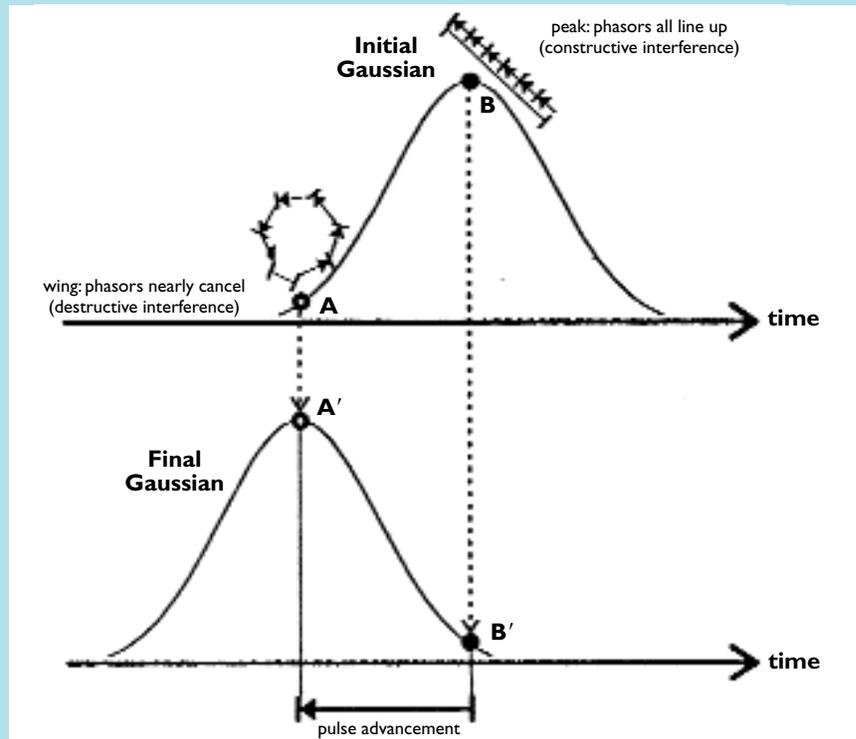
Anomalous group velocities have also been observed, for instance, in microwave waveguides. In one experiment, Mozart’s 40<sup>th</sup> Symphony was encoded on a microwave and transmitted across a tunnel barrier with a group velocity of  $4.7c$  (Ref. 8). While such a graphic demonstration of faster-than- $c$  transmission might at first thought be unsettling, it does not, for the reasons discussed, violate Einstein causality.<sup>5</sup>

So-called Bessel beams have recently been reported to exhibit “superluminal” behavior.<sup>9</sup> Without going into any details, we note here that a Bessel beam can be decomposed into plane waves that intersect a  $z$  axis at an angle  $\theta$ , so that the points of intersection with the  $z$  axis move with the velocity  $c/\cos \theta$ . The  $z$  axis therefore appears to “light up” superluminally, but this is just a geometrical effect and does not imply superluminal signal propagation. The situation here is analogous to the motion of a spot of light made by shining a rotating flashlight on a distant wall. We can imagine the spot moving superluminally, but there would be no violation of Einstein causality because the spot at one instant is not the source of the spot at a later instant.

Equation (1) implies that the group velocity can be made very *small* if  $dn/d\omega$  is large and *positive*, which can occur in the case of electromagnetically induced transparency (EIT).<sup>10</sup> EIT is a consequence of a quantum-mechanical effect in which two



**Figure 3.** Incident (a) and transmitted (b) pulses for a propagation length  $L$  and a group velocity  $v_g > c$ . The shaded portion of the transmitted pulse is completely determined by the shaded portion of the incident pulse.

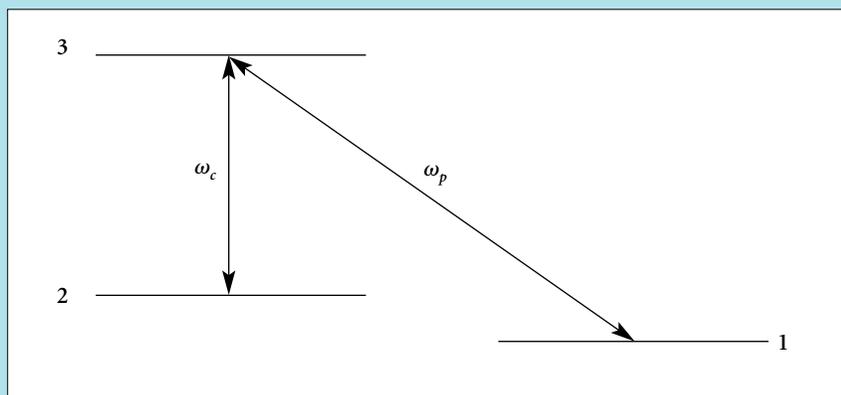


**Figure 4.** Phasor description of group velocity greater than  $c$ .

possible absorption processes can, in effect, interfere destructively with one another in such a way that absorption at a particular frequency does not occur. In Fig. 5 we indicate three states of an atom with two possible absorptive transitions, 1→3 and 2→3. The presence of a coupling field at the frequency  $\omega_c$  can lead to EIT at the probe frequency  $\omega_p$ . In other words, the probe field can propagate without absorption, even though it would be strongly absorbed in the absence of the coupling field.

Associated with EIT is a very large  $dn/d\omega$ , and consequently a very small group velocity, and furthermore  $n$  is unity and the group velocity dispersion is zero

at line center.<sup>11</sup> The extremely small group velocities possible in EIT were first observed in a Bose-Einstein condensate of sodium atoms.<sup>12</sup> Group velocities as small as 17 m/s were observed, as well as the spatial compression of the EIT pulse by the factor  $c/v_g$  without a change in the peak amplitude of the pulse; a pulse that would be 750 m long in free space was observed to be compressed to about 42  $\mu\text{m}$  along the direction of propagation in the ultra-cold gas. This compression is a result of energy being transferred from the front part of the pulse to the atoms and the coupling field, and then being returned to the pulse at its back part. Group velocities of about 90 m/s have been observed in EIT in



**Figure 5.** A “three-state-atom” with allowed transitions  $1 \leftrightarrow 3$  and  $2 \leftrightarrow 3$ . In the presence of a coupling field of frequency  $\omega_c$ , a probe field of frequency  $\omega_p$  can propagate without absorption.

$^{85}\text{Rb}$  at 360 K (Ref. 13), and an extremely narrow nonlinear magneto-optical resonance has been employed to obtain 8 m/s group velocities in room temperature  $^{85}\text{Rb}$  vapor.<sup>14</sup>

If the coupling field does not vary too rapidly in time, the atoms and the probe

field can be described by a single “polariton” field that is a mixture of atom and probe field variables. The proportions of field and atom components in this polariton field are given by  $\cos \theta$  and  $\sin \theta$ , respectively, where the “mixing angle”  $\theta$  depends on the coupling field. The group ve-

locity of the probe field in this description is given by  $c \cos^2 \theta$ , and can be varied in time by varying the coupling field. Thus, if  $\theta$  is changed from 0 to  $\pi/2$ , the group velocity should change from  $c$  to 0; in this case, the polariton goes from being purely photonic to being atomic, and the probe pulse should be effectively halted and coherently stored in the medium. Changing  $\theta$  back to 0 should result in a “re-acceleration” of the probe pulse as the polariton changes from purely atomic to purely photonic.<sup>15</sup>

This remarkable halting, storage, and re-acceleration of light pulses has been observed in sodium vapor at a temperature of about  $1\mu\text{K}$  (Ref. 16) and in rubidium vapor above room temperature.<sup>17</sup> In both experiments, the light storage lasted for a relatively long time, about a millisecond, because the “coherence time” is determined by that of the non-allowed transition  $1 \leftrightarrow 2$  in Fig. 5 rather than the much shorter coherence time associated with an allowed transition. More recently such effects have been observed in a solid using a third field, in addition to probe and coupling fields, to produce the required large value of  $dn/d\omega$  that is ordinarily prevented by the large spectral widths of solids.<sup>19</sup>

It is not inconceivable that the studies of “fast” and “slow” light described in this article will lead to various applications. It has been demonstrated, for instance, that negative group delays can also be realized in relatively simple electronic circuits; the peak of an output pulse exiting a circuit can occur before the peak of the input pulse entering the circuit.<sup>18</sup> It should also be possible to observe similar effects in the case of beams of atoms rather than light.

The slowing and halting of light pulses might find applications in areas such as interferometry or quantum information science, where various quantum processing protocols require a transfer of quantum state information between photons and atoms, and the ability to store this information without loss of phase coherence.<sup>12,20</sup>

Regardless of potential applications, these studies have, in our opinion, added significantly to the understanding of a most basic aspect of optical science.

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