Time Crystals in Optics

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Optical techniques could help take “time crystals”—examples of temporal order and symmetry-breaking analogous to solid-state crystals in space—out of the lab and into real-world use.
In Wilczek’s thought experiment, a superflow of bosons in a tiny ring obeys continuous time-translation symmetry, but solitons could form and, in the presence of a small magnetic-like field, would break this symmetry, spontaneously forming a periodic structure in time—a time crystal.

Temporal counterparts of solid-state crystals

Solid-state crystals—be they salt, sugar, ice or diamond—consist of spatially ordered atoms and molecules. Yet while they appear to embody high symmetry, such spatial crystals actually are fundamentally a form of symmetry-breaking. That’s because the most complete symmetry belongs to empty space, in which no direction merits any preference. When a spatial crystal emerges, continuous space-translation symmetry breaks into discrete symmetry; the perfect symmetry of empty space is replaced with the less symmetric, periodic crystal lattice. As Philip W. Anderson wrote almost exactly 50 years ago in “More is different” [Science 177, 393 (1972)]:

Built from a substrate of atoms and space according to laws which express the perfect homogeneity of space, the crystal suddenly and unpredictably displays an entirely new and very beautiful symmetry. The general rule, however, even in the case of the crystal, is that the large system is...
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less symmetrical than the underlying structure would suggest: Symmetrical as it is, a crystal is less symmetrical than perfect homogeneity.

If one were to envision crystals in the time domain, then, a first requirement would be that such crystals break time-translation symmetry over a long timescale and in the limit of many interacting particles, in the way that solid-state crystals break space-translation symmetry in the thermodynamic limit. And one would also expect temporal analogs of other aspects of condensed-matter spatial crystals. These include the creation of the crystalline structure out of interacting building blocks (atoms or molecules in the case of spatial crystals); the preservation of “long-range order” and structure across infinitely many lattice constants; robustness over a range of environmental parameters; and a phase transition—a drastic change in a suitably defined parameter exemplifying order that requires sufficiently strong interactions between particles.

With these criteria as background, Wilczek proposed the concept of TCs—a system breaking continuous time-translation symmetry in its lowest-energy or ground state—in a celebrated paper in Physical Review Letters. He proposed that attractively interacting bosons in a one-dimensional ring threaded by a proper magnetic-like flux would form a periodically moving, shape-preserving solitary wave (soliton) in the system’s lowest possible energy state, spontaneously creating a periodic crystalline structure in time. (An ordinary oscillator would be at rest in its lowest energy state.) The quantum spreading of the soliton can be avoided when an infinite number of bosons are present, thereby addressing the criteria of long-range order and robustness. And Wilczek referred to the breaking of time-translation symmetry in TCs as occurring through a kind of phase transition for sufficiently strong interactions between bosons.

Contrary to Wilczek’s expectation of periodic motion, however, it turns out that, for an infinite number of bosons, the soliton comes to a complete stop in the ground state. Indeed, “no-go theorems” soon followed which proved that, in the ground state or in equilibrium, periodic motion is absent in many-body systems with quite generic two-body interactions. The notion of time crystals—at least as originally articulated—appeared dead.

A way out: Discrete time crystals

The no-go theorems could have marked the end of time crystals. Yet as definitions were clarified, other possibilities emerged.

One such prospect is TCs that break discrete (rather than continuous) time-translation symmetry. Such symmetry exists in quantum systems that are periodically driven, because the Schrödinger equation is invariant

<table>
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<tr>
<th>Solid-state (spatial) crystal</th>
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<td><strong>CRYSTAL BUILDING-BLOCKS</strong></td>
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<td><strong>SYMMETRY-BREAKING</strong></td>
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<td>Continuous, perfect translation symmetry of empty space replaced by discrete symmetry of crystal unit cells along primitive lattice vectors</td>
<td>Continuous time-translation symmetry, or discrete time-translation symmetry of driven system, spontaneously replaced by a different stable symmetry or larger periodicity</td>
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<td><strong>LONG-RANGE ORDER</strong></td>
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<tr>
<td>Crystalline order maintained over arbitrarily long distance</td>
<td>Periodicity maintained over arbitrarily long time</td>
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<td><strong>ROBUSTNESS</strong></td>
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<tr>
<td>Can exist in a range of environmental parameters</td>
<td>Can exist in a range of system/environmental parameters; no fine-tuning needed</td>
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<td><strong>PHASE TRANSITION</strong></td>
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<td>Dramatic change in an order parameter in space</td>
<td>Dramatic change in an order parameter in time</td>
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when the time parameter is shifted by the period of the driving force. As the temporal symmetry is established by a time-varying external force, the system will be out of equilibrium. The discrete time-translation symmetry will be violated if, in steady state, the dynamics of a system observable evolve periodically with a larger periodicity than the driving period—or, equivalently, oscillate at a subharmonic frequency of the drive frequency. If other crystallization criteria such as long-range order, robustness and phase transition are also fulfilled, the result is a discrete time crystal (DTC).

Period-doubling DTCs were initially proposed in ultra-cold atoms bouncing off of a periodically oscillating atomic mirror, and later in spin-based solid-state systems. Experimentally, they were first observed in coupled-spins systems with a string of trapped ions, in diamond impurities, and later with cryogenic qubits in quantum computers (including a particularly widely publicized recent example using Google’s quantum machine). In these experiments, the lifetimes of the DTCs were of the order of some hundreds of the driving cycle.

Spin-based DTC experiments have thus far been bound to period-doubling time evolution; in a simple picture, two spin flips bring the system back to its initial state. Yet realizing and investigating temporal analogs of solid-state crystals—effects such as Anderson and many-body localization or topological insulators—consisting of many elementary cells, will require “big” DTCs in which symmetry-breaking is dramatic, with periods much longer than twice the driving period. The interacting “particles” in such TCs could, for example, be photons in a nonlinear medium or atoms in a Bose-Einstein condensate. From this vantage point, optics could offer a rich ground for the demonstration and study of big DTCs—a point we will return to a bit later.

**DTCs in closed versus open systems**

The DTCs discussed thus far are periodically driven closed quantum systems, with vanishingly small coupling to the environment and unitary temporal evolution. While the energy of such systems is not constant, because the Schrödinger equation depends explicitly on time, the average change of the system energy when DTCs form is zero. This is quite important, because a generic periodically driven many-body system will eventually heat up to an infinite-temperature state at which no regular time evolution can be observed.

DTCs can also form in open, dissipative systems. Coupling to the environment enables the desired system cooling; on the other hand, it also acts as a noise source which can eventually lead to loss of coherence and decay of regulated TC dynamics. Realizing open-system DTCs thus requires treating the environment as an ally, by balancing the energy pumped to the systems via periodic forcing with the energy released to the environment through ambient coupling. This balance can give rise to periodically evolving steady states that also break the discrete symmetry of the drive. Such dissipative systems are interesting for future practical applications of TCs, because complete isolation is difficult in the real world.

Two recent experiments demonstrated dissipative DTCs. In one, a Bose-Einstein condensate of ultracold atoms was prepared in an optical cavity and pumped by a laser beam. Above a critical pump intensity, the system underwent a superradiant phase transition and the atoms self-organized into one of two possible
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“checkerboard” density patterns stabilized by dissipation. Modulation of the pump strength then periodically modulated the light–matter coupling.

This periodic driving resulted in the system switching between the two checkerboard density patterns with a period twice as long as the driving period—thus creating a period-doubling DTC. Leakage of the atoms from the cavity limited the DTC lifetime to about 15 driving periods.

An all-optical dissipative DTC

The other recent experimental demonstration of a dissipative DTC—involving the interaction of photons in a nonlinear medium—took place recently in our lab. In this far simpler, room-temperature system, a high-\(Q\) (quality factor) magnesium fluoride whispering-gallery-mode cavity was driven simultaneously by two independent lasers, the frequencies of which were chosen to differ by some cavity free spectral ranges. Each laser was self-injection-locked to a target cavity mode of interest, and two different modes of the same modal family were pumped.

This system could generate coherent optical frequency combs whose repetition rates were smaller than the beat note between the two pumps. These spontaneously generated subharmonics signified breaking of the discrete time-translation symmetry—and the possible creation of a DTC.

Confirming the creation of a time crystal requires revisiting the temporal analogs of spatial crystals. The ordered structure of a spatial crystal, for example, is revealed through the observation of the periodic distribution of atoms in space at a fixed moment of time (that is, at the moment of detection)—for example, when an X-ray crystallographic diffraction pattern is captured. With a time crystal, the roles of space and time are swapped; a detector placed at a fixed position in space monitors whether the probability of clicking of the detector behaves periodically in time.

Looking at the time dependence of photon count probability (proportional to the optical field intensity) in the photonic DTC platform, the system possesses discrete time-translation symmetry because of the beating of the two pump lasers. The nonlinear interaction of the photons in the high-\(Q\) resonator spontaneously gives rise to a train of sharply peaked pulses (dissipative cavity solitons), with a period that can be an integer multiple of the drive periodicity. This breaks the underlying time-translation symmetry.

The effect occurs in the limit of infinitely many interacting particles (photons in the nonlinear cavity); it is robust with respect to environmental and system parameters; and it carries on for a very long time compared to the drive periodicity. The emergence of such symmetry-breaking states in the system is marked by subharmonic generation and clear signatures in the cavity transmission. Therefore, the system meets the earlier-articulated criteria for TCs, including long-range order, robustness, and a distinct phase transition.

This relatively simple optical system offers several advantages which highlight the potential of photonics for investigating TC properties. For instance, it supports the realization of DTCs which are big in the

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In one recent DTC demonstration, an assemblage of ultracold atoms, pumped transversely by a laser, formed a checkerboard density pattern (top); periodic driving of the system through modulation of the pump strength (center) led to spontaneous oscillation between two checkerboard patterns at twice the driving period (bottom).

Photonic time crystals

An intriguing alternative view of TCs are so-called photonic time crystals (PTCs), described in a recent review article [E. Gallifi et al., Adv. Photon. 4, 014002 (2022)]. PTCs differ from DTCs in a crucial respect: Their formation is not related to the spontaneous breaking of time-translation symmetry, and thus they do not meet the principal self-organization criterion articulated by Wilczek. Nonetheless, they have properties that could prove interesting and useful in some areas of study.

A PTC involves a homogeneous dielectric medium whose refractive index or other parameters vary periodically in time due to an external driving force. The crystals have allowed and forbidden gaps in wave-number. To create a PTC, a dielectric medium must be modulated in time with a frequency comparable to the frequency of the probe wave propagating in the medium to observe sizable effects. This makes experimental observation of PTCs challenging.

If they can be practically realized, PTCs could be employed to mimic dynamic solid-state optical effects in a photonic platform. For example, PTCs could prove useful in revealing properties in time similar to those of solid-state topological insulators. They could also enable investigation of transport in disordered media. Disorder in PTCs is temporal; thus one can look for phenomena similar to Anderson localization in the time domain using PTCs, as can also be done with DTCs formed by massive particles.

time dimension—that is, which evolve with periodicities much longer than the driving period. Moreover, the platform allows observation of time-crystalline structures with defects (temporal counterparts of solid-state crystal defects such as vacancies, dislocations and interstitials). And the lifetime of DTCs in this optical system is in principle infinite—the DTCs persist as long as the resonator is pumped.

Other time-crystal families

As the definition of TCs has evolved and grown more precise, new TC types have been introduced. Among them are quasi-crystals, fractional crystals and boundary time crystals. A key distinguishing feature of all of them, beyond the breaking of time-translation symmetry, is the fact that robust, long-lived crystalline order can prevail in the limit of infinitely many interacting particles.

A spatial quasi-crystal is a solid-state system in which the distribution of atoms lacks any space-translation symmetry, but that still reveals long-range order because the atoms are not distributed randomly. Quasi-crystals can also emerge in time due to the spontaneous breaking of discrete time-translation symmetry in periodically driven ultra-cold atoms. Such many-body systems can also form so-called fractional DTCs, in which the ratio of the period of motion to the driving period is a rational number. It will be interesting to see if these phenomena can be realized in optical systems.

In another family of TCs, so-called boundary time crystals (BTCs), a macroscopic subset (boundary) of the infinitely many interacting particles in the system (the bulk) breaks the continuous time-translation symmetry in the thermodynamic limit. These BTCs have similarities to monochromatically pumped dissipative cavity solitons in a Kerr resonator, which can be understood when photon count probability is the observable of choice. The drive of the system is a continuous-wave laser (a coherent state) and the probability of counting photons in the optical path is constant (roughly Poissonian statistics). Upon the emergence of solitons from the resonator, a periodic photon count probability is observed, breaking the continuous time-translation symmetry of the drive.

Time-crystalline behavior may also be seen in some familiar optical phenomena.
that demonstrate the breaking of time-translation symmetry. One example is parametric down-conversion, which can occur either in bulk or resonant media with quadratic nonlinearity. Parametric down-conversion carries many of the characteristics of DTCs: the interaction of photons at frequency $\omega$ in a nonlinear medium creates photons at the subharmonic frequency $\omega/2$, a continuous-wave signal whose period is double that of the drive. This process is subject to phase matching and has a threshold.

Whether parametric down-conversion involves the creation of a TC, and what type, however, requires further scrutiny. If the pump is depleted, the system response consists only of the signal photons at the subharmonic frequency, so that both the drive and response possess continuous time-translation symmetry; thus no symmetry-breaking occurs for the photon count probability. On the other hand, without pump depletion, the beating of the pump and its subharmonic breaks the continuous symmetry into a discrete one. In either case, however, emergence of the half-harmonic tone suggests creation of a DTC.

Taking the next steps

Frank Wilczek initiated the era of TCs by envisioning the spontaneous breaking of continuous time symmetry; another seminal idea, the emergence of discrete time-translation symmetry-breaking in nonequilibrium systems, drove subsequent investigation and demonstration of TCs. But many unexplored avenues remain. For instance—in analogy with space crystals and condensed-matter concepts—one might seek temporal equivalents of different phases and phase transitions in the TC context.

Realization of condensed-matter phases in the time domain is a natural next stage in TC development, with a great chance for future applications. Conversely, by examining known optical phenomena through the lens of TCs, novel behavior of photonic systems could be predicted.

Formation of a TC amounts to the spontaneous creation of a clock. But will TCs enable more precise clocks? In the case of the optical dissipative DTCs discussed earlier, the precision of the periodic evolution is limited by that of the external drive. The fact that DTCs evolve with subharmonic frequencies could be used to create efficient high-order frequency dividers where phase noise is significantly suppressed.

Perhaps most intriguingly, photonically enabled TCs could ultimately help solve difficult problems in the solid-state, spatial domain. Condensed-matter effects can sometimes be emulated in much simpler, more compact optical systems. By making the correspondence between a solid-state effect and its optical surrogate more accurate, optical systems may be able to more precisely contribute to investigating open questions in condensed-matter physics. Thus the study of TCs and their properties could spur not only new applications, but also new knowledge in basic physics.

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References and Resources


For complete references and resources, go online: optica-opn.org/link/time-crystals.
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Article References and Resources